the limits of variation of the parameter s for different values of γ_1 and γ_2, and also the variation of Ra_f^* as a function of these parameters.

LITERATURE CITED

1. I. G. Shaposhnikov, Prikl. Mat. Mekh., 17, No. 5, 604 (1953).
2. D. A. Frank-Kamenetskii, Diffusion and Heat Exchange in Chemical Kinetics, Plenum Publ. (1969).
3. D. I. Shvartsblatt, Prikl. Mat. Mekh., 32, No. 2, 276 (1968).
4. V. N. Bloshenko, Zh. Fiz. Khim. 11, No. 12, 3011 (1967).
5. G. Z. Gershuni and E. M. Zhukhovitskii, Convective Stability of Incompressible Fluids, Halsted Press (1976).
6. A. C. Belozerskii, Metal Carbonyls [in Russian], Metallurgizdat, Moscow (1958).
7. H. Schafer Chemical Transport Reactions, Academic Press (1964).

MOTION OF GAS BUBBLES IN AN INFINITE VOLUME OF STATIONARY LIQUID IN A GRAVITATIONAL FIELD

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The resisting forces, velocities, and shape parameters of gas bubbles rising in an infinite volume of liquid are found analytically.

The laws of motion of gas bubbles relative to a liquid are fundamental for the construction of a theory of two-phase media [1]. Ordinarily in the theoretical description of the laws of motion of bubbles in a liquid it is assumed that the bubbles are spheres of radius a and that their motion in the liquid is potential and satisfies the boundary-value problem [2]

$$\Delta\varphi = 0; \quad \mathbf{u} = \operatorname{grad}\varphi; \quad u_r = 0 \quad \text{at} \quad r = a; \quad \mathbf{u} \to \mathbf{U} \quad \text{as} \quad r \to \infty. \tag{1}$$

The solution of problem (1) determines the behavior of the normal u_r and tangential u_η velocity components in the neighborhood of a bubble:

$$u_r = U\left[1 - \left(\frac{a}{r}\right)^3\right]\sin\eta; \quad u_\eta = -U\left[1 + \frac{1}{2}\left(\frac{a}{r}\right)^3\right]\cos\eta, \tag{2}$$

and the pressure distribution on the surface of a bubble is described by Bernoulli's equation

$$\frac{\rho}{2}[u_\eta^2]_{r=a} + p_0 = \text{const}. \tag{3}$$

Equations (2) and (3) are solved for gas bubbles satisfying the pressure balance condition

$$p_0 = p_\text{b} - \frac{2\sigma}{R}. \tag{4}$$

It follows from the condition of static equilibrium (4) that the bubbles can be spherical either if they are very small, when the second term is large, or are stationary with respect to the liquid. In other cases the nonuniformity of the pressure distribution over the surface of a bubble described by Eqs. (2) and (3) must lead to its deformation into an ellipsoid flattened in the direction of motion [2], and to an increase in the area of the interface and consequently to an increase in the dissipative forces as the bubble moves through a viscous liquid.

Thus, to find the laws of motion of a gas bubble it is necessary to solve the problem of the flow of an ellipsoidal bubble and the effect of its velocity with respect to the liquid on its shape.

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In an elliptical coordinate system with its origin at the center of mass of a bubble

$$x = c\,\mathrm{ch}\,\xi \cos\eta \cos\Psi; \quad y = c\,\mathrm{ch}\,\xi \cos\eta \sin\Psi; \quad z = c\,\mathrm{sh}\,\xi \sin\eta. \tag{5}$$

As $\xi \to \infty$, $c \to 0$ these equations approach those for spherical coordinates, where $0 \le \xi \le \infty$, $0 \le \eta \le \pi$, $0 \le \psi \le 2\pi$. For axisymmetric motion the boundary-value problem (1) reduces to the solution of the equations

$$\frac{d^2Y(\xi)}{d\xi^2} + \mathrm{th}\,\xi \frac{dY(\xi)}{d\xi} + \lambda Y(\xi) = 0, \tag{6}$$

$$\frac{d^2R(\eta)}{d\eta^2} - \mathrm{tg}\,\eta \frac{dR(\eta)}{d\eta} - \lambda R(\eta) = 0, \tag{7}$$

where $Y(\xi)$ and $R(\eta)$ are components of the velocity potential

$$\varphi(\xi, \eta) = R(\eta)\,Y(\xi), \tag{8}$$

and λ is the separation constant, which can be found from the expression

$$\lambda = n(n+1); \quad n = 1, 2, 3, \ldots. \tag{9}$$

Making the changes of variables

$$i\,\mathrm{sh}\,\xi = p, \tag{10}$$

$$\sin\eta = k, \tag{11}$$

and taking account of (9), Eqs. (6) and (7) take the conventional form of Legendre equations

$$(1-p^2)\frac{d^2Y}{dp^2} - 2p\frac{dY}{dp} + \lambda Y = 0; \tag{12}$$

$$(1-k^2)\frac{d^2R}{dk^2} - 2k\frac{dR}{dk} + \lambda R = 0. \tag{13}$$

The general solutions of these equations are sums of Legendre functions $P_n(x)$ and $Q_n(x)$ of the first and second kinds, respectively. The potential $\varphi(\xi, \eta)|_{\xi=0}$ must be bounded, the flow is potential, the zonal harmonic $R_n(k)$ cannot change sign more than once in the interval $0 \le \eta \le \pi$, and therefore

$$\lambda = 2; \quad n = 1, \tag{14}$$

$$R_n(k) = k. \tag{15}$$

Taking account of (14), the general solution of Eq. (12) takes the form

$$Y(p) = A_1 p + B_1\left(p \ln\sqrt{\frac{1+p}{1-p}} - 1\right). \tag{16}$$

The coefficients A_1 and B_1 are found from the boundary condition at the surface of a bubble

$$\left.\frac{dY}{dp}\right|_{p=p_0} = 0$$

and the condition that the velocity of the bubble is U at infinity. After satisfying these conditions and taking account of (10), (11), and (15), the general solution for the velocity potential (8) is given by

$$\varphi(\xi, \eta) = -\frac{Uc\sin\eta}{A}\left[\left(\frac{\mathrm{th}\,\xi_0}{\mathrm{ch}\,\xi_0} + \mathrm{arctg\,sh}\,\xi_0\right)\mathrm{sh}\,\xi - \mathrm{sh}\,\xi\,\mathrm{arctg\,sh}\,\xi - 1\right], \tag{17}$$

and the normal u_ξ and tangential u_η velocity components by

$$u_\xi = -\frac{U\sin\eta}{A\sqrt{\mathrm{ch}^2\xi - \cos^2\eta}}\left[\left(\frac{\mathrm{th}\,\xi_0}{\mathrm{ch}\,\xi_0} + \mathrm{arctg\,sh}\,\xi_0\right)\mathrm{ch}\,\xi - \mathrm{ch}\,\xi\,\mathrm{arctg\,sh}\,\xi - \mathrm{th}\,\xi\right]; \tag{18}$$

$$u_\eta = -\frac{U\cos\eta}{A\sqrt{\mathrm{ch}^2\xi - \cos^2\eta}}\left[\left(\frac{\mathrm{th}\,\xi_0}{\mathrm{ch}\,\xi_0} + \mathrm{arctg\,sh}\,\xi_0\right)\mathrm{sh}\,\xi - \mathrm{sh}\,\xi\,\mathrm{arctg\,sh}\,\xi - 1\right]. \tag{19}$$

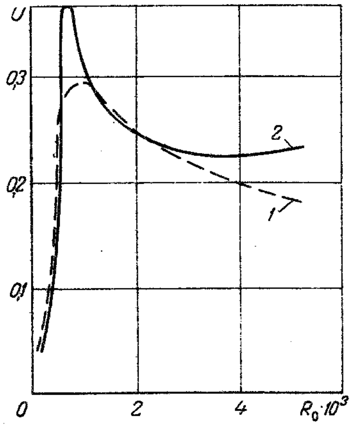

Fig. 1. Rate of rise of a bubble as a function of its radius; (U is the rate of rise of a bubble, m/sec; $R_0 = c \cosh \xi_0 \sqrt[3]{\tan \xi_0}$ is the equivalent radius of the bubble, m); 1) theoretical dependence of rate of rise on radius; 2) experimental dependence of rate of rise on radius from data in [1, 3].

To save writing in Eqs. (17)-(19) we have introduced the notation

$$A = \operatorname{arctg} \operatorname{sh} \xi_0 + \frac{\operatorname{th} \xi_0}{\operatorname{ch} \xi_0} - \frac{\pi}{2}.$$

It follows from (18) and (19) that as $\xi_0 \to \infty$ and $c \to 0$, $c \cosh \xi_0 \approx c \sinh \xi_0 = a$; i.e., when the bubble approaches a spherical shape the velocity of the liquid at its surface

$$\lim_{\substack{c\to 0\\ \xi\to\infty}} u_{\eta 0} = -\frac{3}{2} U \cos \eta; \quad \lim_{\substack{c\to 0\\ \xi\to\infty}} u_{\xi 0} = 0$$

agrees with the known expressions (2). The resistance F_μ to the motion of a bubble in a viscous liquid [2] can be calculated from (17)-(19).

By taking account of (19) the energy E dissipated as a result of viscous effects in the axisymmetric potential flow of a liquid

$$-\frac{dE}{dt} = 2\mu \int_s u_{\eta 0} \frac{\partial u_{\eta 0}}{\partial n} ds \tag{20}$$

can be written in the form

$$-\frac{dE}{dt} = \frac{4\pi\mu c U^2 \operatorname{th} \xi_0}{A^2 \operatorname{ch} \xi_0} \left[\frac{1}{\operatorname{sh}^2 \xi_0} + \frac{1 - \operatorname{sh}^2 \xi_0}{\operatorname{sh}^3 \xi_0} \operatorname{arctg} \frac{1}{\operatorname{sh} \xi_0} \right]. \tag{21}$$

The dissipative force F_μ acting on a gas bubble is given by the expression

$$F_\mu = -\frac{1}{2} \frac{\partial}{\partial U} \left(\frac{dE}{dt} \right) = \frac{4\pi\mu c U \operatorname{th} \xi_0}{A^2 \operatorname{ch} \xi_0} \left[\frac{1}{\operatorname{sh}^2 \xi_0} + \frac{1 - \operatorname{sh}^2 \xi_0}{\operatorname{sh}^3 \xi_0} \operatorname{arctg} \frac{1}{\operatorname{sh} \xi_0} \right]. \tag{22}$$

The rate of rise of a bubble U for steady motion can be determined by equating the force F_μ to the buoyant force F_A:

$$F_\mu = F_A, \tag{23}$$

where

$$F_A = (\rho_L - \rho_g) g V \approx \rho_L g V.$$

The simultaneous solution of Eqs. (22) and (23) determines the rate of rise U of a bubble

$$U = \frac{c^2 \rho_L g A^2 \operatorname{ch}^4 \xi_0}{3\mu F}. \tag{24}$$

Here

$$F = \frac{1}{\operatorname{sh}^2 \xi_0} + \frac{1 - \operatorname{sh}^2 \xi_0}{\operatorname{sh}^3 \xi_0} \operatorname{arctg} \frac{1}{\operatorname{sh} \xi_0}.$$

The shape parameters of a gas bubble c and ξ_0 are determined from an analysis of Eq. (4). In the plane of axial symmetry the boundary of a bubble is an ellipse whose radius of curvature is

$$R_e = \frac{c(\operatorname{ch}^2 \xi_0 - \cos^2 \eta)^{3/2}}{\operatorname{ch} \xi_0 \operatorname{sh} \xi_0}. \tag{25}$$

Since the flow of the liquid is assumed potential, the pressure p_1 at the surface is found from Bernoulli's equation

$$p_1 = p_0 - \frac{\rho u_{\eta 0}^2}{2}. \tag{26}$$

By taking account of (25) and (26) Eq. (4) can be put in the form

$$\sigma \left[\frac{1}{R} - \frac{2}{R_0} \right] = \rho \frac{u_{\eta 0}^2}{2}. \tag{27}$$

Here R_0 is the radius of a spherical bubble of volume V. After substituting Eqs. (25), (19), (24), and the condition for a constant bubble volume into (27), it follows that

$$R_0 = c \operatorname{ch} \xi_0 \sqrt[3]{\operatorname{th} \xi_0}, \tag{28}$$

and the relation between the shape parameters c and ξ_0 takes the form:

$$c^5 = \frac{18\mu^2 \sigma F^2 [\operatorname{ch}^2 \xi_0 \sqrt{\operatorname{th} \xi_0} - \operatorname{sh}^2 \xi_0 (2 - \sqrt{\operatorname{th} \xi_0})]}{\rho^3 g^2 A^2 \operatorname{ch}^5 \xi_0 \sqrt[3]{\operatorname{th} \xi_0}}. \tag{29}$$

The rate of rise of a bubble U calculated by Eq. (24) as a function of its dimensions determined by Eqs. (28) and (29) is plotted in Fig. 1.

The experimental values of the rate of rise of a bubble as a function of its volume are taken from [1, 3]. The agreement of calculated and experimental data is satisfactory for a range of Reynolds numbers

$$0 \leqslant \mathrm{Re} = \frac{2c \operatorname{ch} \xi_0 U \rho}{\mu} \leqslant 2500.$$

The difference between the theoretical and experimental values for Re > 2500 can be explained by the separation of the boundary layer in the afterpart of the bubbles, which reduces their deformation resulting from the nonuniformity of $u_{\eta 0}$.

NOTATION

φ	is the velocity potential;
U	is the rate of rise of bubble;
u_r, u_η	are the normal and tangential velocity components;
p_0	is the pressure in liquid at bubble surface;
p_b	is the gas pressure in bubble,
σ	is the surface tension;
R_0	is the radius of spherical bubble;
μ	is the viscosity of liquid;
ρ_L, ρ_g	are the densities of liquid and gas;
V	is the volume of bubble.

LITERATURE CITED

1. G. Wallis, One-Dimensional Two-Phase Flows, McGraw-Hill, New York (1969).

2. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs (1962).
3. B. N. Yudaev, O. V. Tsirlin, and A. A. Yushkin, "Steady motion of a gas bubble," Inzh.-Fiz. Zh., No. 4, 25 (1973).

MAGNETORHEOLOGICAL EFFECT NEAR THE CURIE POINT

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Results of an experimental study are presented which pertain to the magnetorheological effect in a ferrofluid at temperatures near the Curie point of the dispersed phase.

An external magnetic field, while structurizing a ferrofluid suspension, radically alters its rheological properties (magnetorheological effect). This effect was experimentally studied under conditions where the magnetic properties of the dispersed ferromagnetic phase could be assumed to be independent of the temperature [1-3]. As is well known, ferromagnetic materials have the fundamental property that the long-range magnetic order breaks down due to heating until it completely vanishes (at the Curie point). This limits utilization of the magnetorheological effect at temperatures near the Curie point. On the other hand, simultaneous action of a magnetic fluid during heating of a magnetorheological fluid to temperatures near the Curie point can be useful for several practical applications.

These authors studied the magnetorheological characteristics of a system with a dispersed phase having its Curie point within the test range of temperatures. Other components of the active medium were selected on the basis of a low-temperature sensitivity of their physicochemical properties over the test range of temperatures, so as to ensure stability of the system during the entire period of time needed for performing the experiment.

The saturation magnetization of our ferrofluid suspension during changes of the temperature was measured by the Faraday method. From the thus obtained curves depicting the temperature dependence was found the critical point corresponding to the loss of magnetic properties by the substance. This critical point was 145°C. The critical temperature in a weak magnetic field (H = 2 Oe) was somewhat higher and equal to 158°C.

Rheological measurements were made with a rotary viscometer "Rheostat-2", its nonmagnetic working part placed in a magnetic field normal to the shear plane. The test cell was thermostated over the test range of temperatures, this range extending from room temperature to beyond the critical point established on the basis of magnetization measurements.

The resulting flow curves are rheograms characteristic of magnetorheological systems with a nonlinear dependence of the shearing stress on the strain rate [2].

Heating of this ferrofluid suspension during deformation at a given strain rate lowers its effective viscosity until the magnetorheological effect has been completely compensated. At a temperature near the critical point for this system (t = 145°C) the effective viscosity of the magnetorheological composite material asymptotically approaches some value within the corresponding critical range, while the effect of the magnetic field gradually weakens (Fig. 1).

The temperature dependence of the viscosity component due to interaction between particles of the system upon application of a magnetic field can be examined through the expression

$$\Delta\eta = \frac{\eta_{Ht} - \eta_t}{\eta_{H0} - \eta_0}, \tag{1}$$

where γ_{Ht} denotes the effective viscosity of the ferrofluid suspension in a magnetic field at a given temperature; η_t, viscosity without a magnetic field at that temperature; and $\eta_{H,0}$, η_0, respectively, the effective viscosity in a magnetic field and the viscosity without a magnetic field at the initial temperature of the system.

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